Enterprise Risk Management Through Strategic Allocation of Capital

Joint Work
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Overview

- Introduction to ERM and Literature Review
- Baseline One-Period Model
- The Choice of Risk Appetite
- A Numerical Illustration
- The Two-Period Model
- Discussions and Conclusion

What is ERM?

The concept of Enterprise Risk Management (ERM)

- Managing risks holistically rather than separately
- Unique features of ERM invovles
  - risk appetite
  - inter-relations between risks
  - risk prioritization
  - alignment of strategic goals and risk considerations
- The users of ERM: corporations, universities, and government

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Value Creation

The ultimate goal of ERM is to create value for stakehoders

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A desired approach

- A framework for operational decisions
- Capture important characteristics
- Flexible and adaptive

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Research Question

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- The dynamic framework allows the firm to account for the changing business environment
- Provides a conceptual framework capable of facilitating more general ERM modeling

ERM Literature

Components of ERM

- Determination of ERM: Liebenberg and Hoyt (2003)
- (Quantile-based) risk measure: Dowd and Blake (2006)
- Risk appetite: Nocco and Stulz (2006)
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Chance-constrained programming approach (Charnes and Cooper, 1959, 1962, 1963)

The Conceptual Framework

To achieve business target in light of a multitude of risk considerations

(1) project risk $\leq$ project risk appetite
(2) financial risk $\leq$ financial risk appetite
(3) operational risk $\leq$ operational risk appetite
(4) hazard risk $\leq$ hazard risk appetite
(5) overall risk $\leq$ overall risk appetite
(6) other considerations (e.g., budget constraint)

- by making appropriate operational decisions

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  - $i = 1, ..., K$ real projects (P) (e.g., K manufacturing product lines)
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- Model assumptions
  - Random returns
  - Direct loss from hazard risk is proportional to total capital
  - Indirect loss from hazard risk is a percentage of direct loss
  - Hazard risk is mitigated by insurance

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Chance Constraints

- Account for four major types of firm risks: project risk, financial risk, operational risk, hazard risk
- Serve as the tool to incorporate the firm's risk appetite and risk prioritization decisions
- Overall liquidity/solvency likelihood constraint specifies the likelihood of being unable to meet obligations and intertwines all the decision variables together

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- Selection is governed by project risk appetite α_1

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- Could incorporate hedging policies in the decision framework (e.g., Caldentey and Haugh 2006)

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- Interactions between project risk and financial risk

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Deterministic Constraints

- Budget constraint:

$$\sum_{i=1}^{K} w_i^{(P)} + \sum_{j=1}^{N} w_j^{(A)} + u(1+d)(1+\theta)\mu \leq 1$$

- Strategic constraint:

$$\sum_{i=1}^{K} w_i^{(P)} \geq \gamma_3$$

- Range constraints:

$$w_i^{(P)} \geq 0, w_j^{(A)} \geq 0, 0 \leq \nu \leq 1$$

The ERM Framework

$$\max_{w_i^{(P)}, w_j^{(A)}, u} \text{E[project returns} + \text{financial returns - insurance]}$$

s.t. P[project returns$\geq$hurdle rate]$\geq$(1-α_1)

P[financial returns$\geq$hurdle rate]$\geq$(1-α_2)

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P[total returns - all risk - insurance$\leq$obligation]$\leq \bar{\alpha}$

budget constraint, strategic constraint, range constraints

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budget constraint, strategic constraint, range constraints

The ERM Framework

$$\max_{w_i^{(P)}, w_j^{(A)}, u} \text{E[project returns + financial returns - insurance]}$$

s.t. P[project returns$\geq$hurdle rate]$\geq$(1-α_1)

P[financial returns$\geq$hurdle rate]$\geq$(1-α_2)

P[op risk$\leq$risk limit]$\leq \alpha_3$

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Computation of the Constraint Set

- To convert from the VaR type risk constraint to a deterministic constraint for use in mathematical programming, we use the fact that if

$$P[X \geq t] \geq 1 - \alpha$$

then

$$t \leq F^{-1}(\alpha)$$

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- So for example the financial risk constraint

$$P[\sum_{j=1}^{N} w_j^{(A)}(1 + r_j^{(A)}) \geq (\sum_{j=1}^{N} w_j^{(A)})(1 + r_0^{(A)})] \geq 1 - \alpha_2$$

becocmes

$$(1+r_0^{(A)})(W^{(A)})^T\iota-(W^{(A)})^T(\iota+E(r)^{(A)}) \leq \Phi^{-1}(\alpha_2)\sqrt{(W^{(A)})^T\Sigma^{(A)}W^{(A)}}$$

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    - determines the ability to raise capital and the cost of capital (West 1973)
    - influences corporate policies and actual business strategies (Sufi 2007)

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- Moody's Rate Transition Matrix

	Rating To:							
Rating From:	**Aaa**	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa-C**	**Default**
Aaa	91.75%	7.26%	0.79%	0.17%	0.02%	0.00%	0.00%	0.00%
Aa	1.32%	90.71%	6.92%	0.75%	0.19%	0.04%	0.01%	0.06%
A	0.08%	3.02%	90.24%	5.67%	0.76%	0.12%	0.03%	0.08%
Baa	0.05%	0.33%	5.05%	87.50%	5.72%	0.86%	0.18%	0.31%
Ba	0.01%	0.09%	0.59%	6.70%	82.58%	7.83%	0.72%	1.48%
B	0.00%	0.07%	0.20%	0.80%	7.29%	80.62%	6.23%	4.78%
Caa-C	0.00%	0.03%	0.06%	0.23%	1.07%	7.69%	75.24%	15.69%

Average one-year rating transition matrix, 1920-2005, conditional upon no rating withdrawal.
Source: Moody's Default and Recovery Rates of Corporate Bond Issuers, 1920-2005, March 2006.

Source: Nocco and Stulz. 2006. Enterprise Risk Management Theory and Practice. J. Corp. Fin.

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Individual risk appetite parameters facilitate the quantification of risk prioritization decisions:

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Risk Prioritization

Individual risk appetite parameters facilitate the quantification of risk prioritization decisions: smaller value indicates higher priority of the risk

Numerical Example: Baseline Setting I

Purposes

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- Demonstrate how to determine risk appetite parameters

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- Hazard risk is independent of the other risk categories

Numerical Example: Baseline Setting II

Description of investment opportunities

Expected Return and Variance				
	R&D Project	Manufacturing Project	Index Fund	3 Month T-Bill
Expected return	0.3	0.1	0.12	0.038
Variance	0.3	0.003	0.03	0.00025

Correlation of the investments				
	R&D Project	Manufacturing Project	Index Fund	3 Month T-Bill
R&D Project	1	0.1	0.05	-0.05
Manufacturing Project	0.1	1	0.05	-0.05
Index Fund			1	-0.1
3 Month T-Bill				1

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- Parameter Value

Risk Appetite					Risk Limits			
α_1	α_2	α_3	α_4	$\overline{\alpha}$	$r_0^{(P)}$	$r_0^{(A)}$	l_{op}	m
0.05	0.05	0.05	0.05	0.0008	0	0	0.2	0.01
Hazard Risk					Op Risk Factors		Strategic Factor	Borrowed Capital
μ	σ	θ	d		γ_1	γ_2	γ_3	c
0.01	0.1	0.1	0.2		0.2	0.1	0.5	0.7

Optimization Results

Optimization Results under the Baseline Setting

Decision Variable	Optimal Value
Investment in the R&D project	0.0558
Investment in the manufacturing project	0.6228
Investment in the index fund	0.0542
Investment in the Treasury bill	0.2546
The proportion of hazard risk insured	0.9479
Optimal Return	1.0806

Risk Prioritization

No Prioritization vs. With Prioritization

Decision Variable	No Prioritization All α's = 0.05	With Prioritization $\alpha_1 = 0.1$, $\alpha_4 = 0.01$
Investment in the R&D project	0.0558	0.0991
Investment in the manufacturing project	0.6228	0.5197
Investment in the index fund	0.0542	0.0647
Investment in the Treasury bill	0.2546	0.3039
The proportion of hazard risk insured	0.9479	0.9625
Optimal Return	1.0806	1.0862

Interactions Between Risks

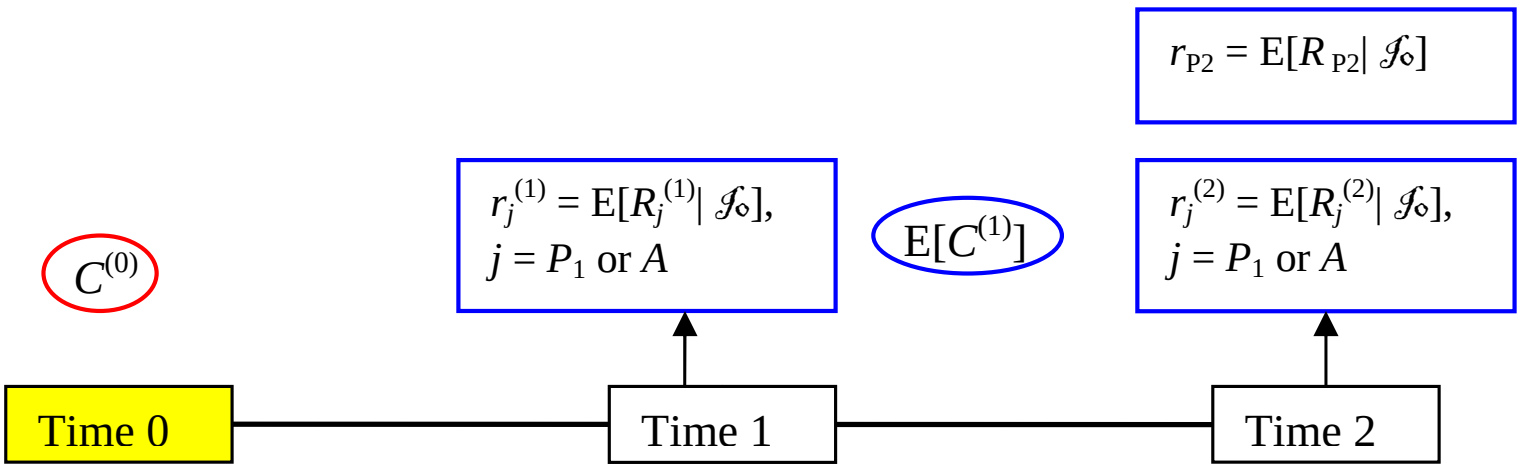
High Op Risk Factors vs. Low Op Risk Factors

Decision Variable	High Op Factors (γ_1=0.2, γ_2=0.1)	Low Op Factors (γ_1=0.05, γ_2=0.05)
R&D project	0.0558	0.0813
Manufacturing project	0.6228	0.9062
Index fund	0.0542	0
Treasury bill	0.2546	0
Hazard risk insured	0.9479	0.9479
Optimal Return	1.0806	1.1004

Two-Period Setting

- Risk/return optimization in a two-period planning horizon
- Three broad types of investment opportunities
  - Short-term (one-period) real projects: invest at the beginning of each period, return at end of each period
  - Long-term (two-period) real projects: invest at the beginning of period 1, return at end of period 2
  - Financial assets

Strategic Time Line



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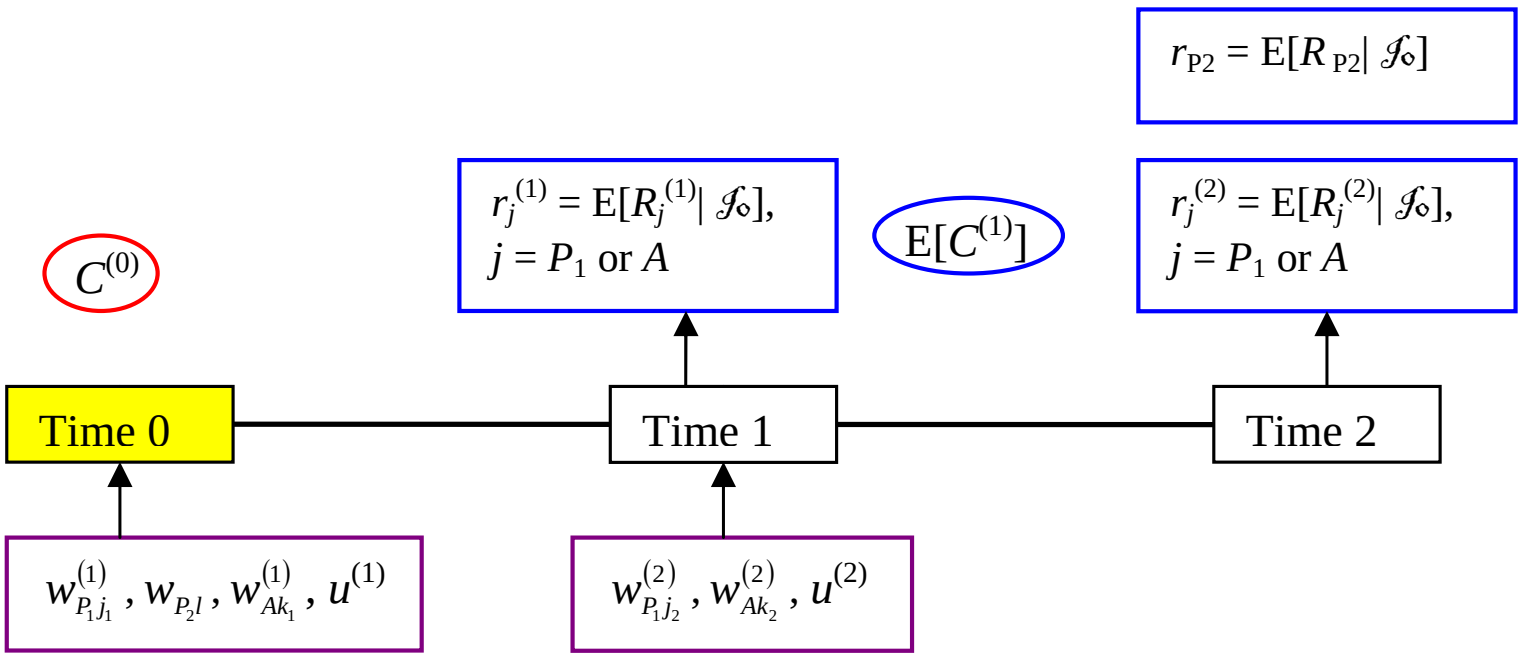

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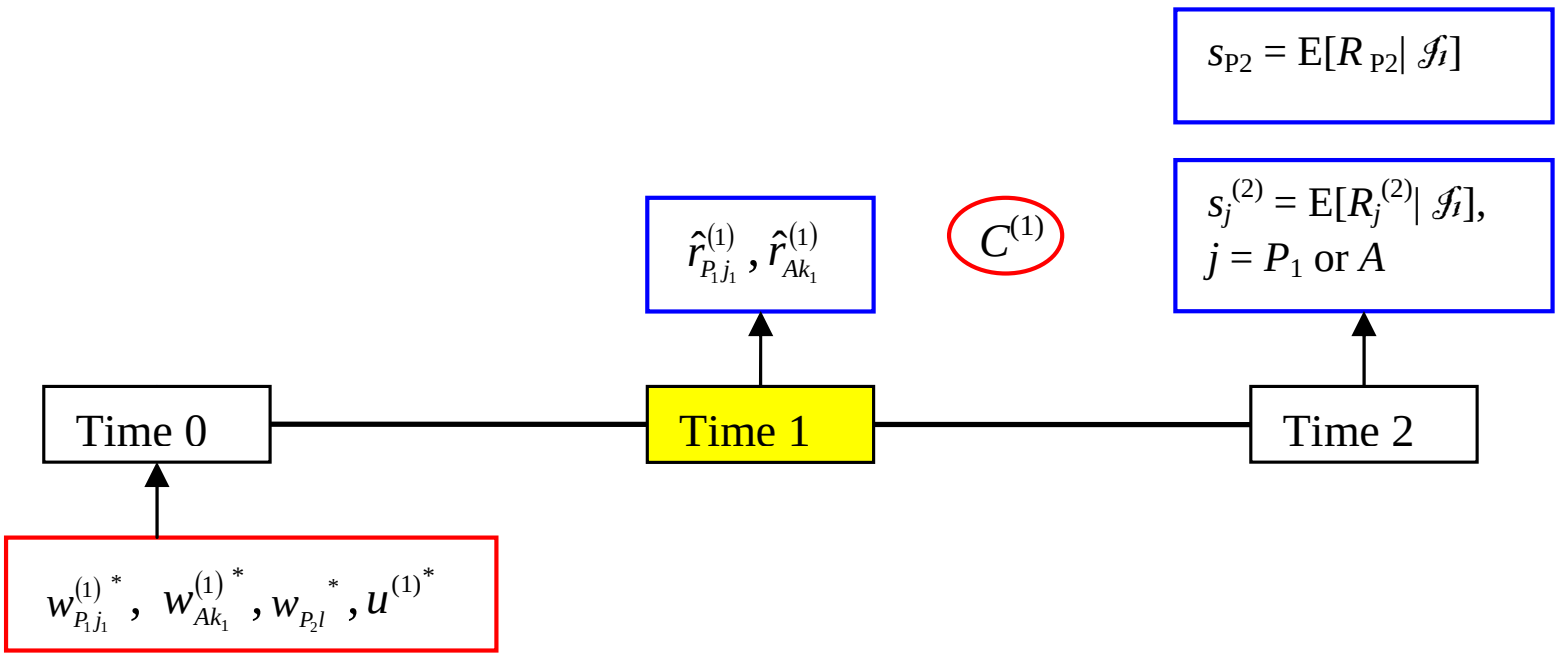

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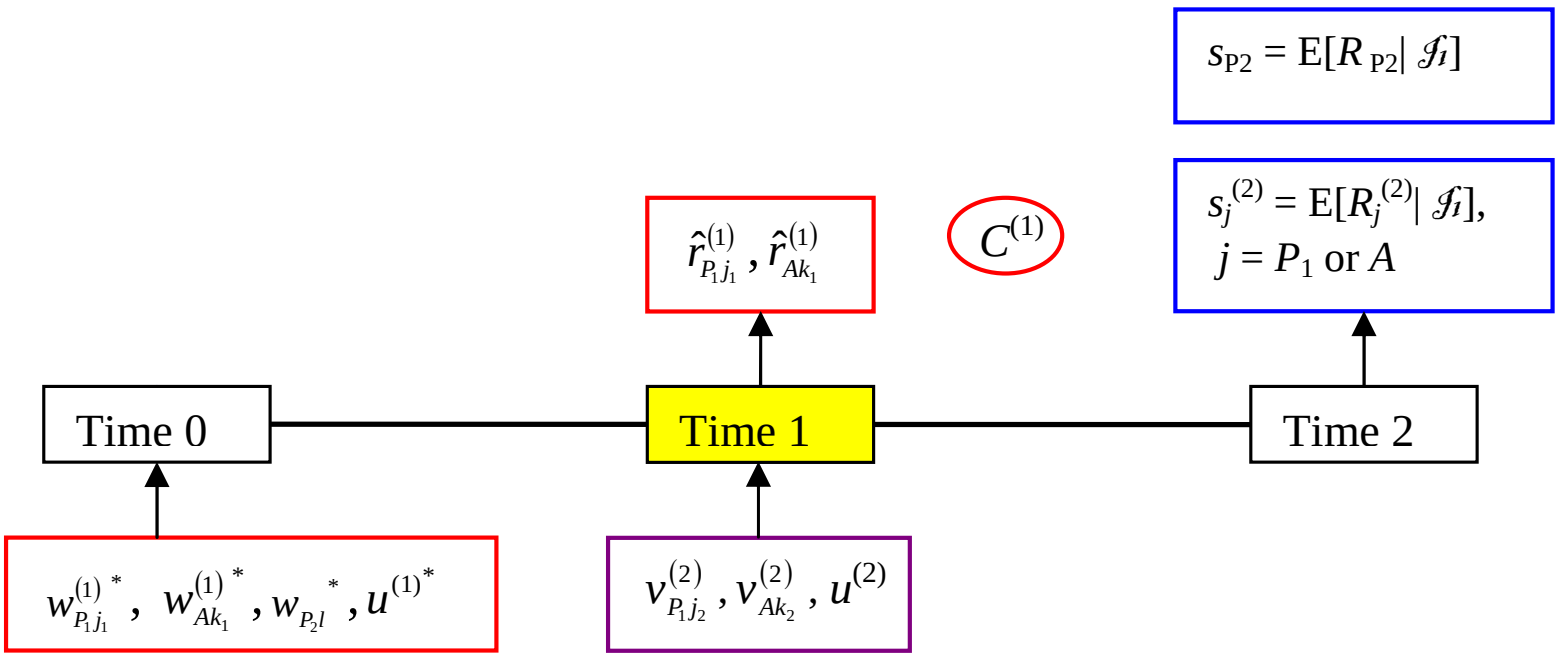

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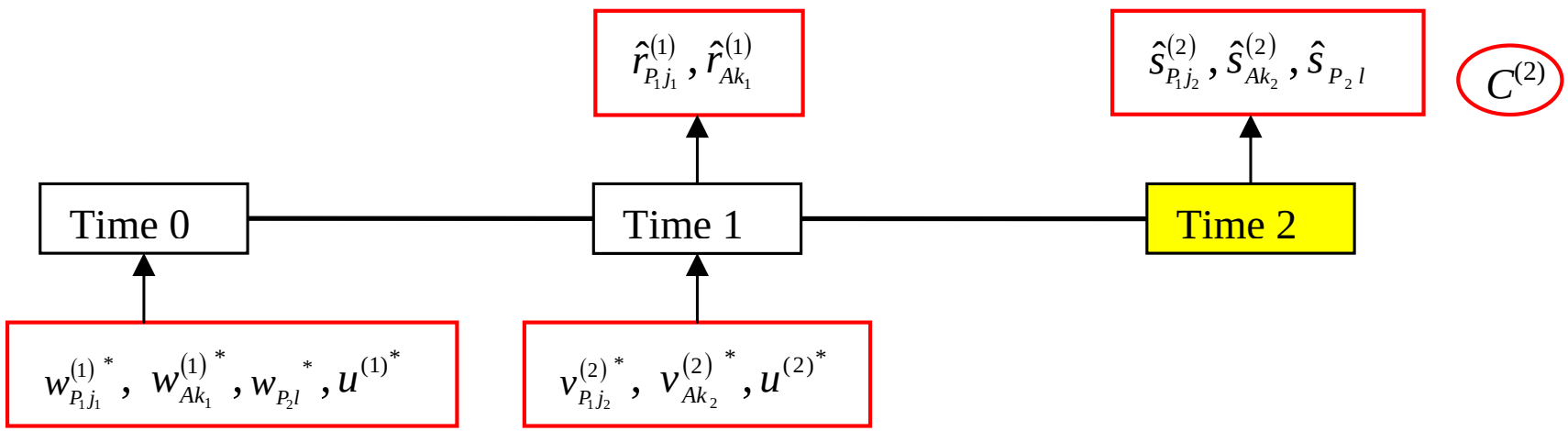

The Dynamic Framework

- Stage 1 (based on information set I_0)

max E[end-of-horizon total return]

s.t. Each risk constraint for period 1

Each risk constraint for period 2

Budget constraint, Strategic constraint, Range constraint

- Stage 2 (based on information set I_1)

max E[end-of-horizon total return]

s.t. Each risk constraint for period 2

Budget constraint, Strategic constraint, Range constraint

Discussions

- Distributional assumptions
  - Elliptically symmetric distribution (convex programming)
  - The copula method (search methods)
- Risk management strategies
- Decision of risk appetite
- Other model extensions
  - Multi-stage model
  - Other stochastic optimization models